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SUBJECT: Reduction of Look Angle Sensitivity to Altitude Updates During LM Descent Visibility Phase - Case 20063-3

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MF7- 4164-13

MEMORANDUM FOR FILE

INTRODUCTION

This paper describes a simple modification to the Lunar Module (LM) powered descent guidance equations which significantly improves astronaut visibility while flying over rough or sloping lunar terrain. This scheme reduces the sensitivity of the look angle to radar altitude updates during the visibility phase by shifting the downrange position of the landing site in proportion to the altitude updates. The landing site adjustments are in a direction such that the apparent motion of the landing site due to sloping terrain is reduced, but not eliminated.

The first portion of the paper discusses the visibility problem and describes the theory of the modification. In the remainder of the paper, a comparison between the modified scheme and the present scheme is presented through look angle profiles for various terrain models.

Work on this study was conducted by R. W. Srch while on rotational assignment in Department 4264.

(NASA-CR-95558) REDUCTION OF LOOK ANGLE SENSITIVITY TO ALTITUDE UPDATES DURING LM DESCENT VISIBILITY PHASE (Bellcomm, Inc.) 34 p

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GENERAL DISCUSSION

During the visibility phase, the LM flies a trajectory as directed by the Quadratic Guidance equation given by

$$\ddot{r}(t) = a + b(T-t) + c(T-t)^2$$

where T is the terminal time, and a, b, and c are computed to satisfy terminal position, velocity, and acceleration constraints. The terminal time T is selected to satisfy the terminal horizontal jerk component.

In addition to guidance commands, the computer also repetitively calculates and displays the look angle* of the current landing site. The Systems Engineer reads the displayed angle and repeats it to the Commander who identifies the landing point by sighting along the corresponding angle etched on the LM window. For landing site visibility, the look angle (λ) must be greater than 25 degrees. However, it is desirable that the look angle remain above 35 degrees. Although the guidance law directs flight along a nominal trajectory which allows the astronauts visual contact with the landing footprint, it has no explicit control over the look

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* Look angle is defined in Appendix A.

angle. Thus changes in the estimated state from nominal will be reflected in the look angle profile. One significant source of fluctuations in the estimated state is the landing radar which is measuring altitude above the local terrain.

The effects of terrain variations on visibility is now explained with reference to Figure 1. For simplicity, a unity weighting function is assumed for the radar altitude measurements. In addition, the problem is discussed as though the updates are applied to the altitude of the landing site rather than the estimated LM altitude. This simplification makes the problem easier to visualize but does not affect the results since the two point boundary problem solved by the guidance equations depends to a first order only upon the relative positions of the vehicle and the landing site.

Figure 1 illustrates the apparent horizontal motion of the landing site when a negative step in terrain is encountered by the LM. Before the step is reached, the guidance and navigation equations would consider the landing site to be at point 1, but the Commander would view point 3 on the lunar surface. Once the step is encountered the equations would update to point 2, and the Commander would view an apparant horizontal shift in the landing site from point 3 to point 2. Since the depression angle (α) is about 15 degrees, the landing site appears to shift by an amount equal to approximately four times the altitude update.

In addition to the apparent motion of the landing site, the look angle is reduced when the negative step is encountered. An initial decrease in λ results from the instantaneous change of the landing site position from point 1 to point 2. This change in the two point boundary value problem results in a decrease in the required vertical acceleration. Since the required horizontal acceleration remains unchanged, a pitch transient occurs to make the thrust vector more horizontal, thus decreasing λ further. The effect of a positive step would be similar with a resulting increase in the look angle. A slope would produce a continual change but the overall effect would be equivalent to a series of steps. Flight over rough lunar terrain would yield a composite situation of erratic shifting of the look angle and the apparent landing site position plus a rough, jostling ride for the astronauts.

ANALYTICAL INVESTIGATION

Examination of the visibility problem indicated that the sensitivity of look angle to altitude updates could be reduced by shifting the downrange position of the landing site to compensate for the change in altitude. Appendix A contains the derivation of an expression for the look angle sensitivity (eq. 30A); and an expression which defines the shift in the downrange position of the landing site in terms of an angle η for zero look angle sensitivity (eq. 33A). The tangent of η is equal to the ratio of the altitude update to the horizontal displacement of the landing site.

For use in this study, the following conditions for a nominal trajectory from High Gate to Hover were used:

Initial Conditions at High Gate

Vertical Velocity	=	-133 ft/sec.
Horizontal Velocity	=	470 ft/sec.
Altitude	=	6,100 ft.
Range to Landing Site	=	24,225 ft.

Desired Conditions at Hover

Vertical Velocity	=	-5.1 ft/sec.
Horizontal Velocity	=	0 ft/sec.
Altitude	=	100 ft.
Horizontal Jerk	=	0.0975 ft/sec^3
Total Acceleration	=	0 ft/sec^2

This trajectory is illustrated by a range versus altitude profile in Figure 2. The attitude of the LM along this trajectory is shown at 5 second intervals.

The expression for zero look angle sensitivity (eq. 33A) was applied to this particular trajectory and the values of the updating angle (η) which satisfy the equation are depicted in Figure 3. This figure shows that the angle η varied between 34 and 30 degrees for the region just beyond High Gate to an uprange distance of approximately 2,000 feet. The mean value for η within these boundaries was calculated to be 31.54 degrees. The deviation from the mean for nearly all values of η was less than 2 degrees. In fact, for a sample of $n = 18$ points within this region representing measurements 5 seconds apart, the mean was 31.54 degrees and the standard deviation was 1.3 degrees. Hence, for this trajectory, the value of 31.54 degrees was selected to define the relationship between the altitude update and landing site displacement.

Note that the ideal situation would be to update along the line-of-sight since the Commander would detect no change in the landing site due to radar altitude updates. For this reason the look angle sensitivity vs. range was compared for three updating angles: $\eta = 31.54°$, $\eta = \alpha$ (depression angle) and $\eta = 90°$. Figure 4 demonstrates the improvement of look angle sensitivity using the 31.54° updating angle compared to either vertical or line-of-sight updating.

An expression for the percent change in thrust for an instantaneous change in altitude was derived and is included in Appendix B. Figure 5 illustrates that the descent scheme using a 31.54° updating angle has a slightly greater thrust sensitivity. (The thrust vs. range profile for this trajectory is included in Figure 6.) However, the figures which follow will show that the tradeoff for this slightly greater thrust sensitivity is a significant improvement in look angle during the visibility phase. The new updating scheme produces deviations in the fuel reserve at hover. These fuel deviations are due to the downrange shift in the landing site associated with this new technique, and are equivalent to the deviations obtained for astronaut redesignation of the landing site.

SIMULATION RESULTS

Using the Bellcomm Apollo Simulation Program, the two schemes (η=31.54° and η=90°) were simulated and compared for flight over various terrain features. An altitude measurement weighting of 1.0 was used to simplify the analysis.

The types of terrain that were examined included step functions of various magnitudes at different points uprange and slopes of two and three degrees. These simulated flights provide information about the effects of step functions and slopes upon look angle profiles for η = 31.54° and η = 90° as compared with that for a nominal trajectory. The look angle profiles, look angle versus time-to-go, are shown in Figures 9 through 13.

If the LM flies over a terrain feature resembling a step, Figures 7, 8 and 9 demonstrate the change in look angle profiles that occur. For steps of 1000 feet, 500 feet, and 100 feet, the new updating scheme produces look angle profiles that lie significantly closer to the nominal than the vertical updating scheme. Also shown are the transients that occur as the abrupt change in terrain occurs. As the radar detects a change in altitude, the line-of-sight changes instantaneously. This is the initial transient in the profiles. Sensing the changes in the two point boundary value problem, the guidance equations then command a change in pitch angle which accounts for the transient during the following guidance cycle. Vertical updating reinforces the transient and increases it to a larger deviation from the nominal. This produces a substantial change in the look angle profile immediately, whereas, the $\eta = 31.54°$ updating scheme does not allow a large deviation to be sustained. An obvious advantage of this updating scheme is that it tends to damp out transients due to abrupt terrain changes and return to the proximity of the nominal. Although the look angle profile drifts away from the nominal toward the later part of the flight, this scheme significantly reduces the effects of terrain change on look angle during the time the astronauts are appraising the acceptability of the landing site.

Figures 10 and 11 provide look angle profiles of terrain slopes of 2 and 3 degree magnitudes, respectively. Whereas the step function introduced a single perturbation which caused the look angle profile to change, the slope continually introduces perturbations with each radar altitude measurement. In both cases, the scheme using $\eta = 31.54°$ provides look angle profiles which are significantly closer to the nominal. In the worst instance, a negative slope, the vertical updating causes the LM to lose visibility sooner and more rapidly than $\eta = 31.54°$. Hence, the time that the astronauts have for appraising the landing site is improved using the modified scheme.

CONCLUSIONS

The results demonstrate that look angle profiles can be improved by shifting the landing site in proportion to the altitude updates. In addition, the scheme reduces the apparent motion of the landing site due to the updates, but does not eliminate it entirely. The modification is extremely simple to implement, adding almost no complexity to the guidance scheme.

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Atts.
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Atts.
References
Appendices A and B
Figures 1 - 12

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2. F. Heap, "A LM Powered Descent Strategy", Bellcomm Memorandum for File, Case 320, September 30, 1966.

3. "Bellcomm Apollo Simulation Program", Operations Manual, Bell Telephone Laboratories, January 1, 1965.

APPENDIX A

Derivation of Look-Angle Sensitivity Equation

Object: To derive an expression such that the change in look angle with respect to an instantaneous change in position is zero.

Definition of Look Angle: the angle between the thrust vector and the line-of-sight vector as shown in Figure 12.

$$\lambda = \theta - \alpha \tag{1A}$$

(see Figure 12)

$$\delta\lambda = \delta\theta - \delta\alpha \tag{2A}$$

but

$$\theta = \tan^{-1}(a_x/-a_z) \tag{3A}$$

$$\alpha = \tan^{-1}(x/-z) \tag{4A}$$

therefore

$$\delta\theta = \frac{a_x \delta a_z - a_z \delta a_x}{a_x^2 + a_z^2} \tag{5A}$$

$$\delta\alpha = \frac{x\delta z - z\delta x}{x^2 + z^2} \tag{6A}$$

From the quadratic guidance (ref. 1)

$$a_x = -\frac{6(\dot{x}+\dot{x}_D)}{T_{go}} - \frac{12(x-x_D)}{T_{go}^2} + g \tag{7A}$$

$$a_z = -\frac{6(\dot{z}+\dot{z}_D)}{T_{go}} - \frac{12(z-z_D)}{T_{go}^2} \tag{8A}$$

then

$$\delta a_x = \frac{\partial a_x}{\partial x}\,\delta x + \frac{\partial a_x}{\partial T_{go}}\,\frac{\partial T_{go}}{\partial z}\,\delta z \tag{9A}$$

$$\delta a_z = \frac{\partial a_z}{\partial z}\,\delta z + \frac{\partial a_z}{\partial T_{go}}\,\frac{\partial T_{go}}{\partial z}\,\delta z \tag{10A}$$

from (7A) and (8A)

$$\frac{\partial a_x}{\partial x} = \frac{\partial a_z}{\partial z} = -\frac{12}{T_{go}^2} \tag{11A}$$

$$\frac{\partial a_x}{\partial T_{go}} = \frac{6(\dot{x}+\dot{x}_D)}{T_{go}^2} + \frac{24(x-x_D)}{T_{go}^3} \tag{12A}$$

$$\frac{\partial a_x}{\partial T_{go}} = -\frac{a_x}{T_{go}} + \frac{12(x-x_D)}{T_{go}^3} + \frac{g}{T_{go}} \tag{13A}$$

$$\frac{\partial a_z}{\partial T_{go}} = \frac{6(\dot{z}+\dot{z}_D)}{T_{go}^2} + \frac{24(z-z_D)}{T_{go}^3} \tag{14A}$$

$$\frac{\partial a_z}{\partial T_{go}} = -\frac{a_z}{T_{go}} + \frac{12(z-z_D)}{T_{go}^3} \tag{15A}$$

We can now write:

$$\delta\lambda = \frac{(a_x \delta a_z - a_z \delta a_x)}{a_x^2 + a_z^2} - \frac{(x\delta z - z\delta x)}{x^2 + z^2} \tag{16A}$$

$$\delta\lambda = \frac{(z\delta x - x\delta z)}{x^2 + z^2} - \frac{12}{T_{go}^2}\frac{(a_x\delta z - a_z\delta x)}{a_x^2 + a_z^2}$$

$$+ \frac{\left(a_x \frac{\partial a_z}{\partial T_{go}} - a_z \frac{\partial a_x}{\partial T_{go}}\right)\frac{\partial T_{go}}{\partial z}\delta z}{a_x^2 + a_z^2} \qquad (17A)$$

$$\delta\lambda = \frac{(z\delta x - x\delta z)}{x^2 + z^2} - \frac{12}{T_{go}^2}\frac{(a_x\delta z - a_z\delta x)}{a_x^2 + a_z^2}$$

$$+ \frac{\left\{\frac{12}{T_{go}^3}\left[a_x(z - z_D) - a_z(x - x_D)\right] + \frac{g}{T_{go}}\right\}\frac{\partial T_{go}}{\partial z}\delta z}{a_x^2 + a_z^2} \qquad (18A)$$

To find $\frac{\partial T_{go}}{\partial z}$, we can make use of the horizontal jerk equation (ref. 1).

$$J_z = \frac{6(\dot{z} + \dot{z}_D)}{T_{go}^2} + \frac{24(z - z_D)}{T_{go}^3} = \text{constant} \qquad (19A)$$

therefore

$$\delta J = 0 = \left[-\frac{12(\dot{z}+\dot{z}_D)}{T_{go}^3} - \frac{72(z-z_D)}{T_{go}^4} \right] \delta T_{go} + \frac{24}{T_{go}^3}\,\delta z \qquad (20A)$$

and therefore:

$$\delta T_{go} = \left[\frac{2}{(\dot{z}+\dot{z}_D) + \dfrac{6(z-z_D)}{T_{go}}} \right] \delta z \qquad (21A)$$

This relation can now be substituted into (18A). We now make a change of coordinates:

let

$$\eta = \tan^{-1}\left(\frac{\delta x}{-\delta z}\right) \qquad (22A)$$

$$\delta r = (\delta x^2 + \delta z^2)^{\frac{1}{2}} \qquad (23A)$$

then

$$\delta x = \delta r \sin \eta$$

$$\delta z = -\delta r \cos \eta$$

and from figure 12:

$$R = (x^2+z^2)^{\frac{1}{2}}$$

$$z = -R \cos \alpha$$

$$x = R \sin \alpha$$

$$a = \left(a_x^2 + a_z^2\right)^{\frac{1}{2}}$$

$$a_z = a \cos \theta$$

$$a_x = a \sin \theta$$

Substituting these relations into (18A) and (20A) we get

$$\delta\lambda = -\frac{1}{R} \sin (\eta-\alpha)\delta r - \frac{12}{aT_{go}^2} \sin (\eta-\theta)\delta r$$

$$+ \frac{2\left[-\frac{12a_x(z-z_D)}{T_{go}^3} + \frac{12a_z(x-x_D)}{T_{go}^3} + \frac{g}{T_{go}}\right] \cos \eta\delta r}{a^2\left[(\dot{z}+\dot{z}_D) + \frac{6(z-z_D)}{T_{go}}\right]} \qquad (24A)$$

let

$$A = -\frac{1}{R} \tag{25A}$$

$$B = -\frac{12}{aT_{go}^2} \tag{26A}$$

$$C = \frac{24\left[-a_x(z-z_D) + a_z(x-x_D) + \frac{gT_{go}^2}{12}\right]}{a^2T_{go}^2\left[6(z-z_D) + (\dot{z}+\dot{z}_D)T_{go}\right]} \tag{27A}$$

then

$$\frac{\delta\lambda}{\delta r} = A \sin(\eta-\alpha) + B \sin(\eta-\theta) + C \cos\eta \tag{28A}$$

for a radar altitude update of δx , we have

$$\frac{\delta\lambda}{\delta x} = \frac{\delta\lambda}{\delta r}\frac{1}{\sin\eta} \tag{29A}$$

or

The sensitivity of λ to altitude updates is:

$$\boxed{\frac{\delta\lambda}{\delta x} = \frac{1}{\sin \eta}\left[A \sin(\eta-\alpha) + B \sin(\eta-\theta) + C \cos \eta\right]} \tag{30A}$$

We want $\frac{\delta\lambda}{\delta x} = 0$ for zero look angle sensitivity,

therefore:

$$A \sin \eta \cos \alpha - A \cos \eta \sin \alpha + B \sin \eta \cos \theta$$

$$- B \cos \eta \sin \theta + C \cos \eta = 0 \tag{31A}$$

or

$$\sin \eta \left[A \cos \alpha + B \cos \theta\right] - \cos \eta \left[A \sin \alpha + B \sin \theta - C\right] = 0$$

(32A)

$$\boxed{\tan \eta = \frac{\sin \eta}{\cos \eta} = \frac{A \sin \alpha + B \sin \theta - C}{A \cos \alpha + B \cos \theta}} \tag{33A}$$

Thus tan η must satisfy eq. (33A) for zero look angle sensitivity.

APPENDIX B

Derivation of Thrust Sensitivity Expression

Object: To obtain an expression for the per cent change in thrust for an instantaneous change in position; that is $\left(\frac{\delta F/F}{\delta x}\right)$ (during quadratic guidance phase).

$$F = ma = \text{thrust} \tag{1B}$$

$$\delta F = m\delta a + a\delta m \tag{2B}$$

However, for an instantaneous change in position, there is an instantaneous change in thrust due only to an instantaneous change in acceleration, that is $\delta F = m\delta a$. Therefore:

$$\frac{\delta F}{F} = \frac{\delta a}{a} \tag{3B}$$

$$a = \left(a_x^2 + a_z^2\right)^{\frac{1}{2}} \tag{4B}$$

assuming $a_y = 0$ for the two-dimensional case. Then

$$\delta a = \frac{1}{2}\left(a_x^2 + a_z^2\right)^{-\frac{1}{2}} \left(2a_x\delta a_x + 2a_z\delta_z\right) \tag{5B}$$

$$\delta a = \frac{1}{a^2}\left(a_x\delta a_x + a_z\delta a_z\right) \tag{6B}$$

The differentials δa_x , δa_z can be evaluated from the quadratic guidance equations given in Appendix A (eqs. (7A), (8A)) to yield

$$\frac{\delta a}{a} = \frac{-12}{a^2 T_{go}^2}\left(a_x\delta x + a_z\delta_z\right)$$

$$+ \frac{+24}{a^2 T_{go}^2}\left\{- \frac{a^2 T_{go}^2}{12} + a_x\left[\left(x - x_D\right) + \frac{gT_{go}^2}{12}\right] + a_z\left(z - z_D\right)\right\}\delta z \tag{7B}$$

let

$$D = \frac{-12}{a^2 T_{go}^2} \tag{8B}$$

$$E = \frac{24}{a^2T_{go}^2}\left\{-\frac{a^2T_{go}^2}{12} + a_x\left[(x-x_D) + \frac{gT_{go}^2}{12}\right] + a_z(z-z_D)\right\} \qquad (9B)$$

then

$$\frac{\delta a}{a} = D(a_x\delta x + a_z\delta z) + E\delta z \qquad (10B)$$

from figure 12 and equations (22A), (23A)

$$a_x = a \sin\theta$$

$$a_z = -a\cos\theta$$

$$\delta x = \delta r \sin\eta$$

$$\delta z = -\delta r \cos\eta$$

then

$$\frac{\delta a}{a} = D\left[(a\sin\theta)(\delta r\sin\eta) + (-a\cos\theta)(-\delta r\cos\eta)\right]$$

$$+ E(-\delta r\cos\eta) \qquad (11B)$$

$$\frac{\delta a}{a} = (aD \sin \theta)\delta r \sin \eta + (aD \cos \theta - E)\delta r \cos \eta \qquad (12B)$$

therefore

$$\frac{\delta a/a}{\delta r} = aD \sin \theta \sin \eta + (aD \cos \theta - E) \cos \eta \qquad (13B)$$

but

$$\frac{\delta a/a}{\delta x} = \frac{\delta a/a}{\delta r} \frac{1}{\sin \eta} \qquad (14B)$$

Thus

$$\boxed{\frac{\delta F/F}{\delta x} = \frac{\delta a/a}{\delta x} = aD \sin \theta + (aD \cos \theta - E) \cot \eta} \qquad (15B)$$

= per cent change in thrust for an instantaneous change in position .

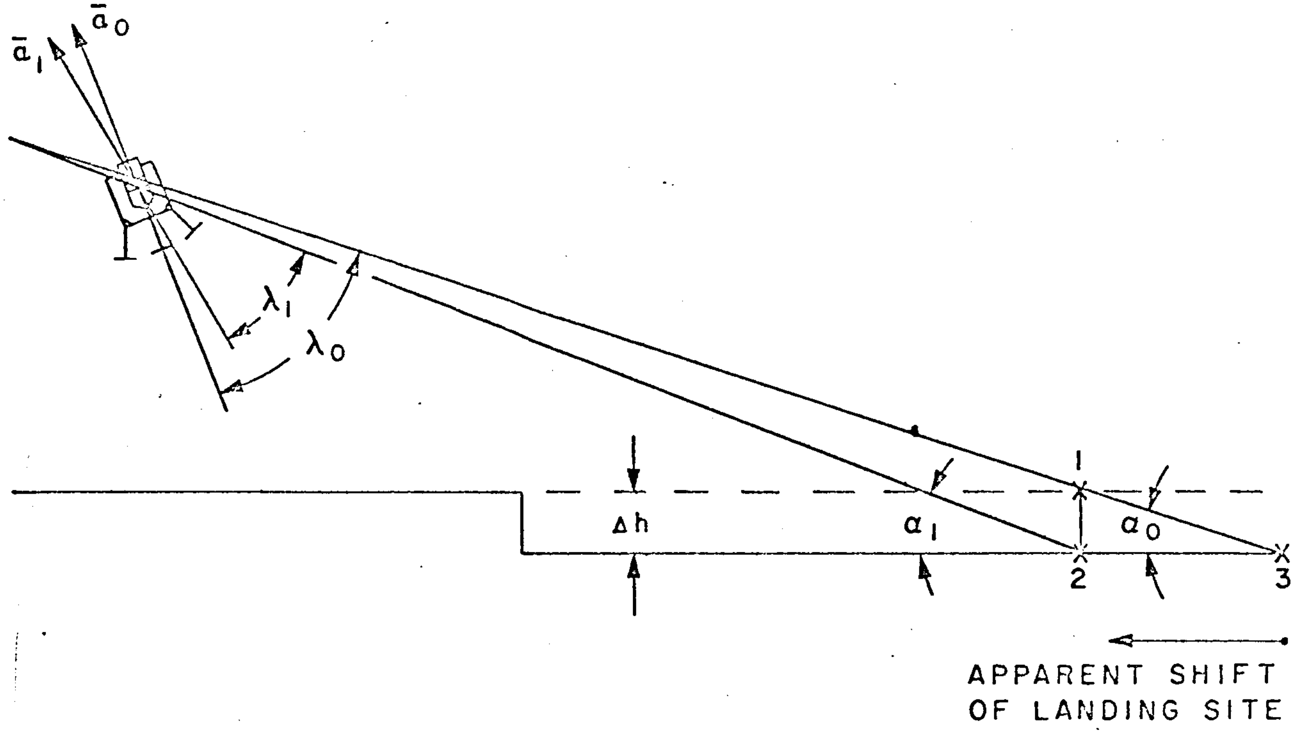

Figure 1.

Apparent shift in landing site and reduction of look angle due to a negative step (Δh) in the lunar terrain.

Point 1: position of L.S. before step is reached (computed by navigation equations)

Point 2: new position of L.S. after navigation equations are updated upon reaching negative step in terrain.

Point 3: position on Lunar surface that commander views as the landing site before step is reached.

α_0, α_1 : depression angle before and after step is reached.

$\bar{a}_0, \bar{a}_1$: thrust acceleration vector before and after step is reached.

λ_0, λ_1 : look angle before and after step is reached.

Δh : magnitude of step in lunar terrain.

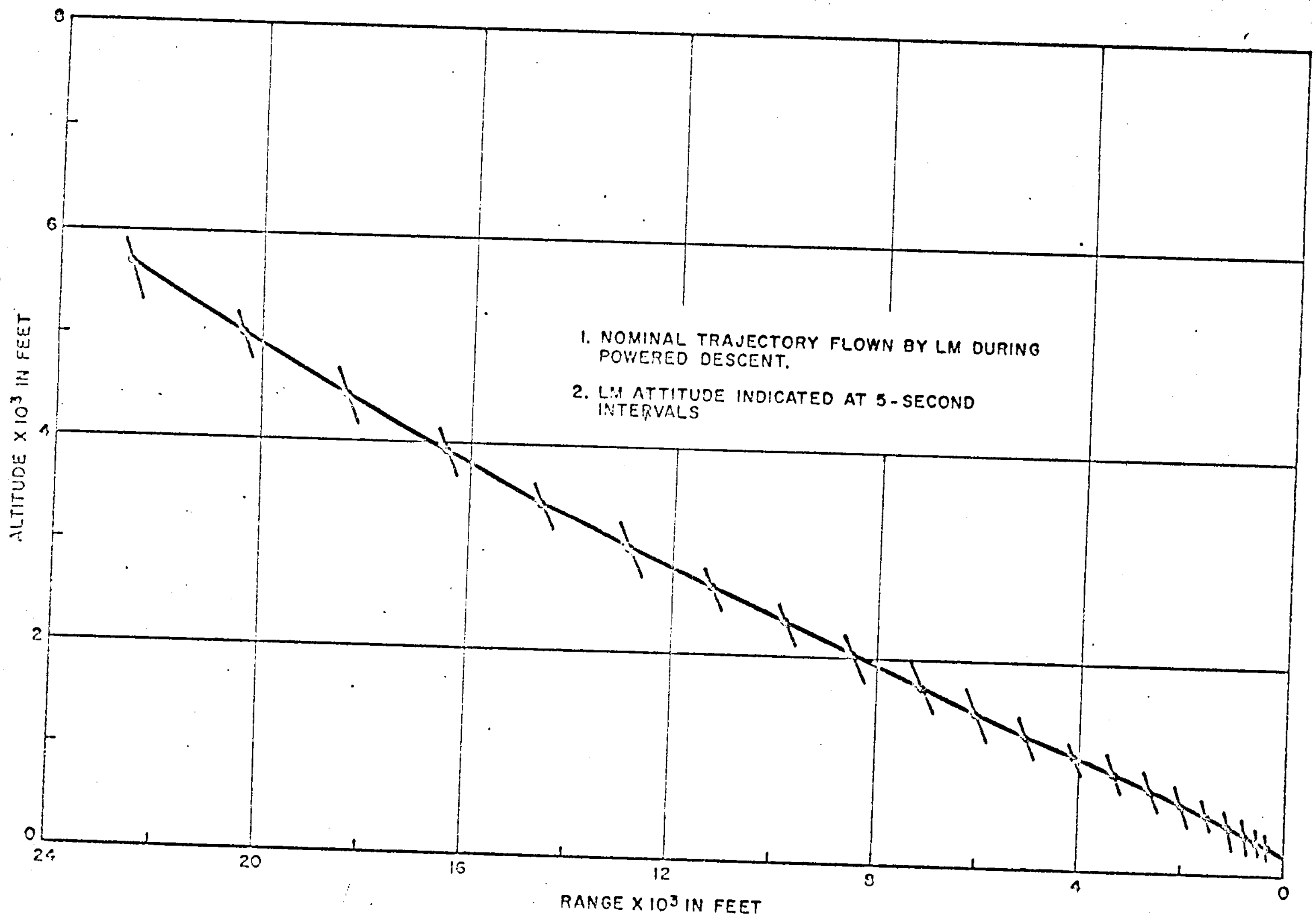

FIGURE 2. RANGE VS. ALTITUDE

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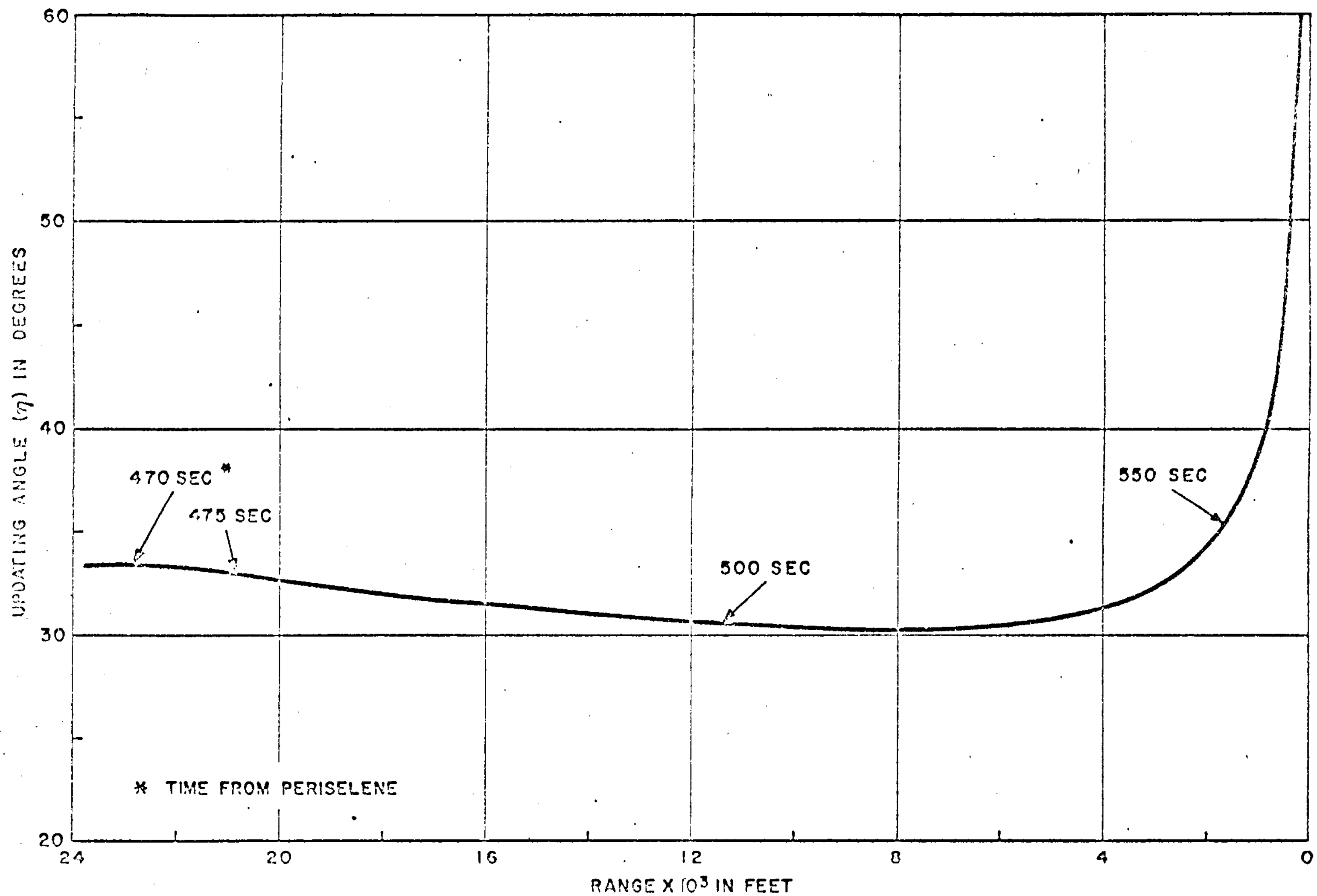

FIGURE 3. UPDATING ANGLE (η) VS RANGE FOR ZERO LOOK ANGLE SENSITIVITY

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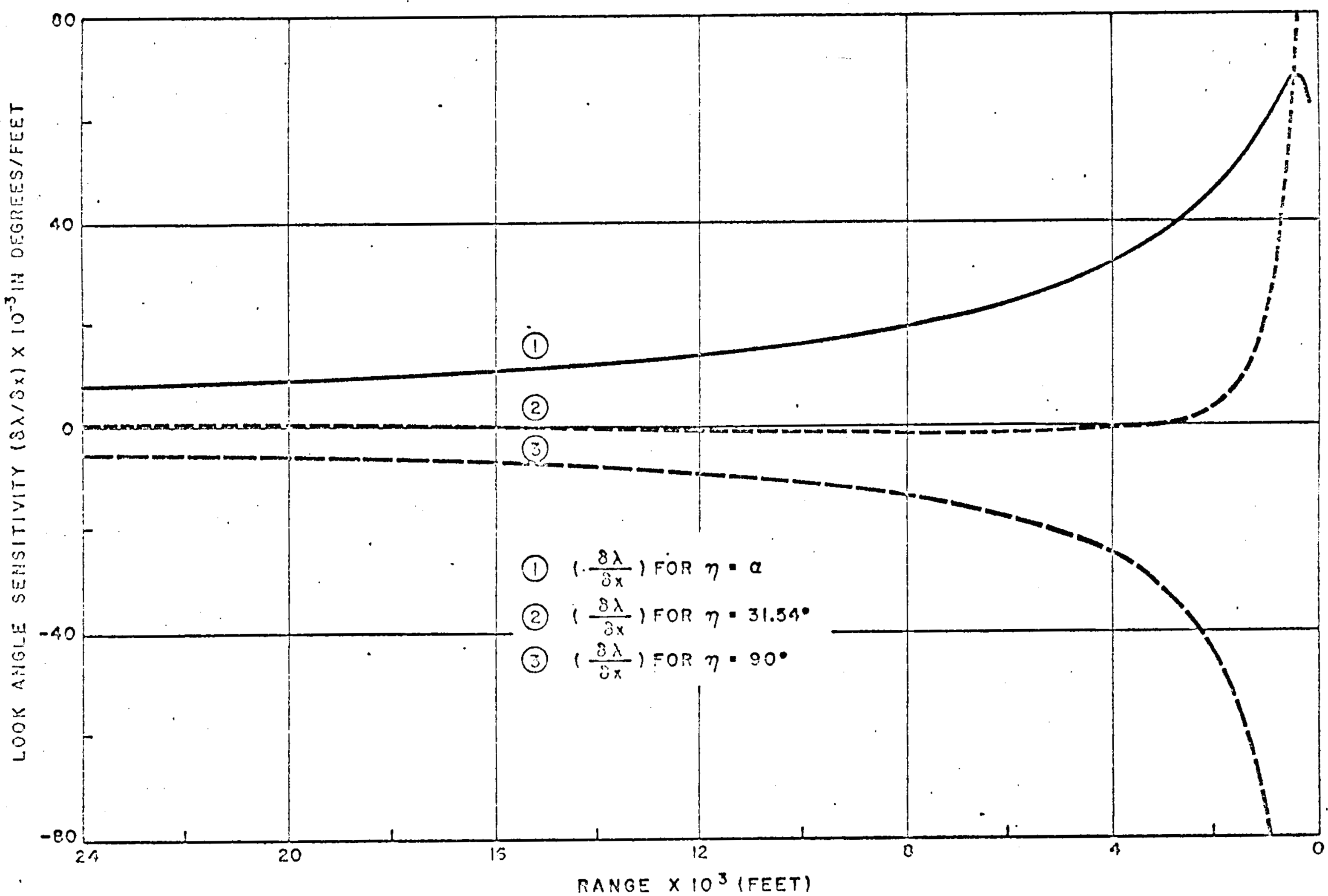

FIGURE 4. LOOK ANGLE SENSITIVITY VS RANGE

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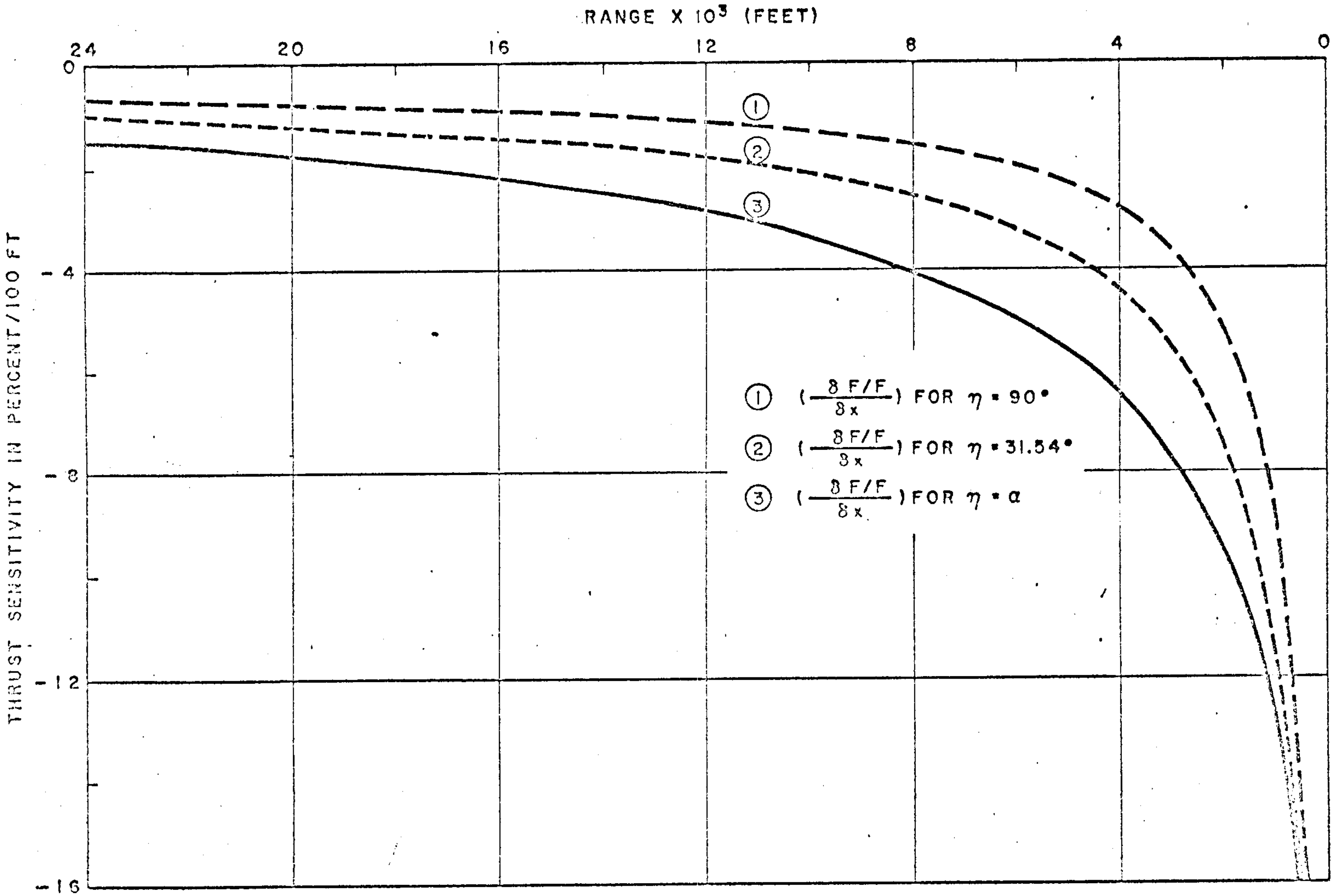

FIGURE 5. THRUST SENSITIVITY VS RANGE

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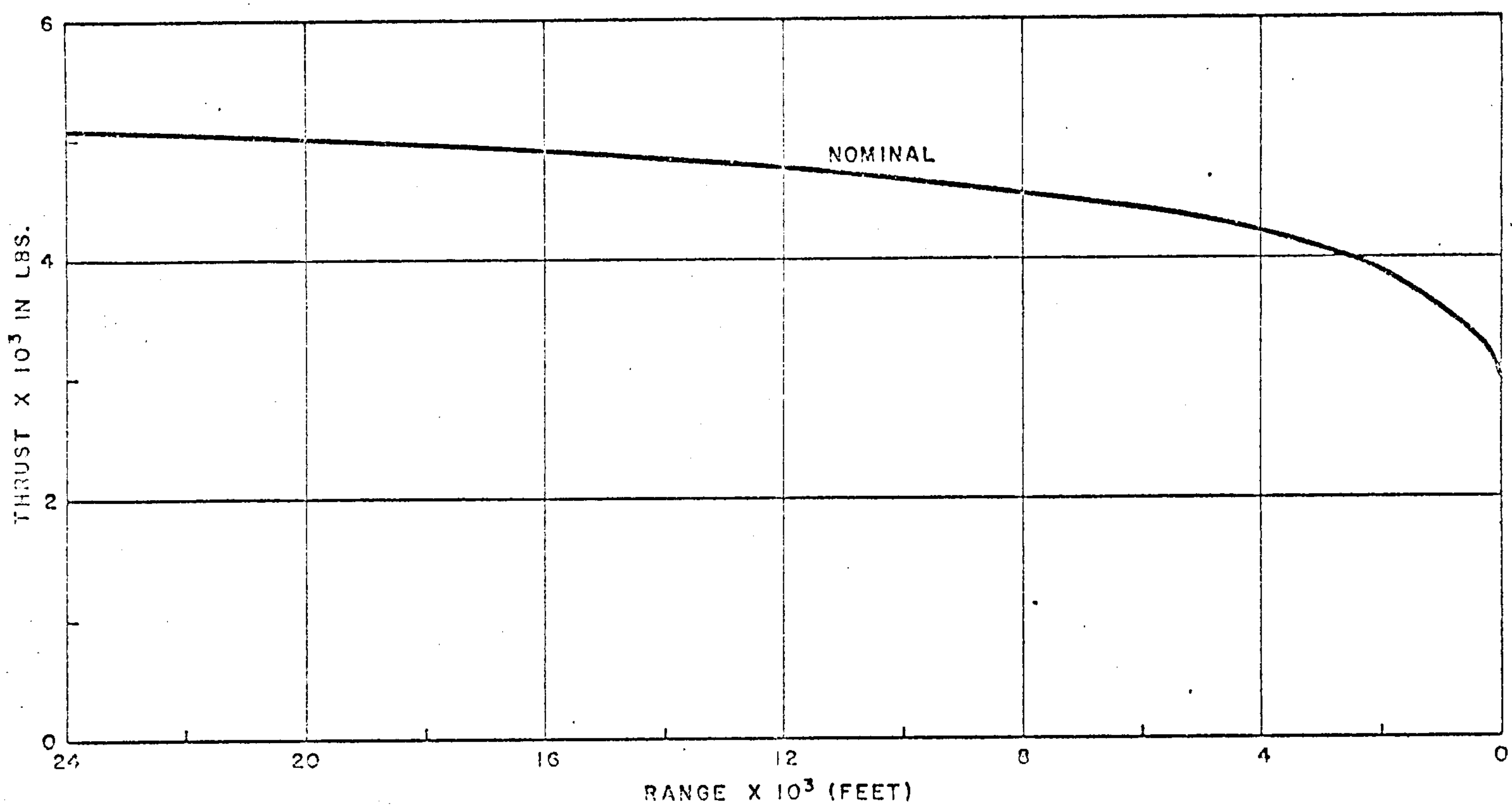

FIGURE 6. THRUST VS RANGE

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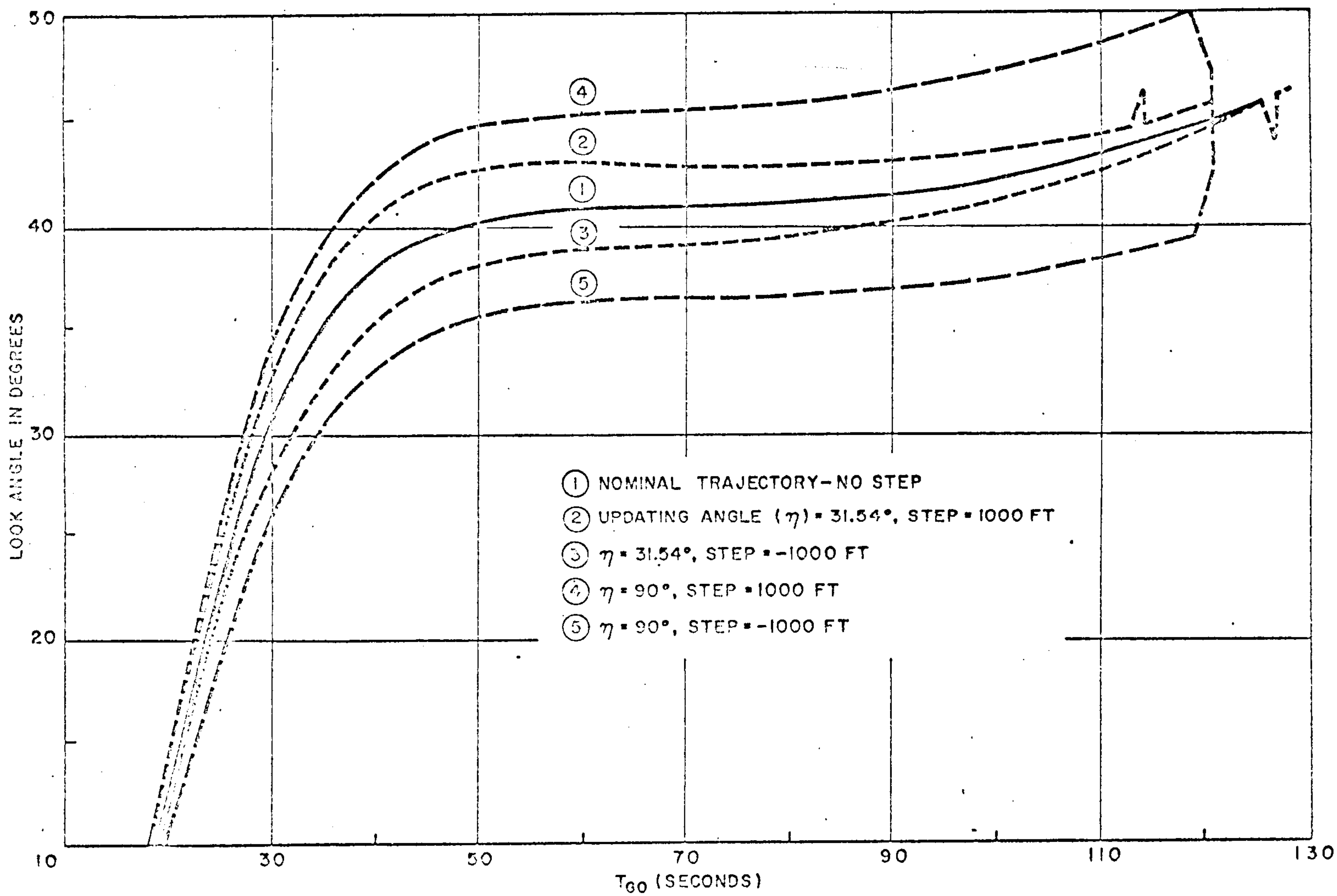

FIGURE 7. LOOK ANGLE VS TIME-TO-GO (STEP OF 1000 FT AT A RANGE 20,000 FT FROM LANDING SITE)

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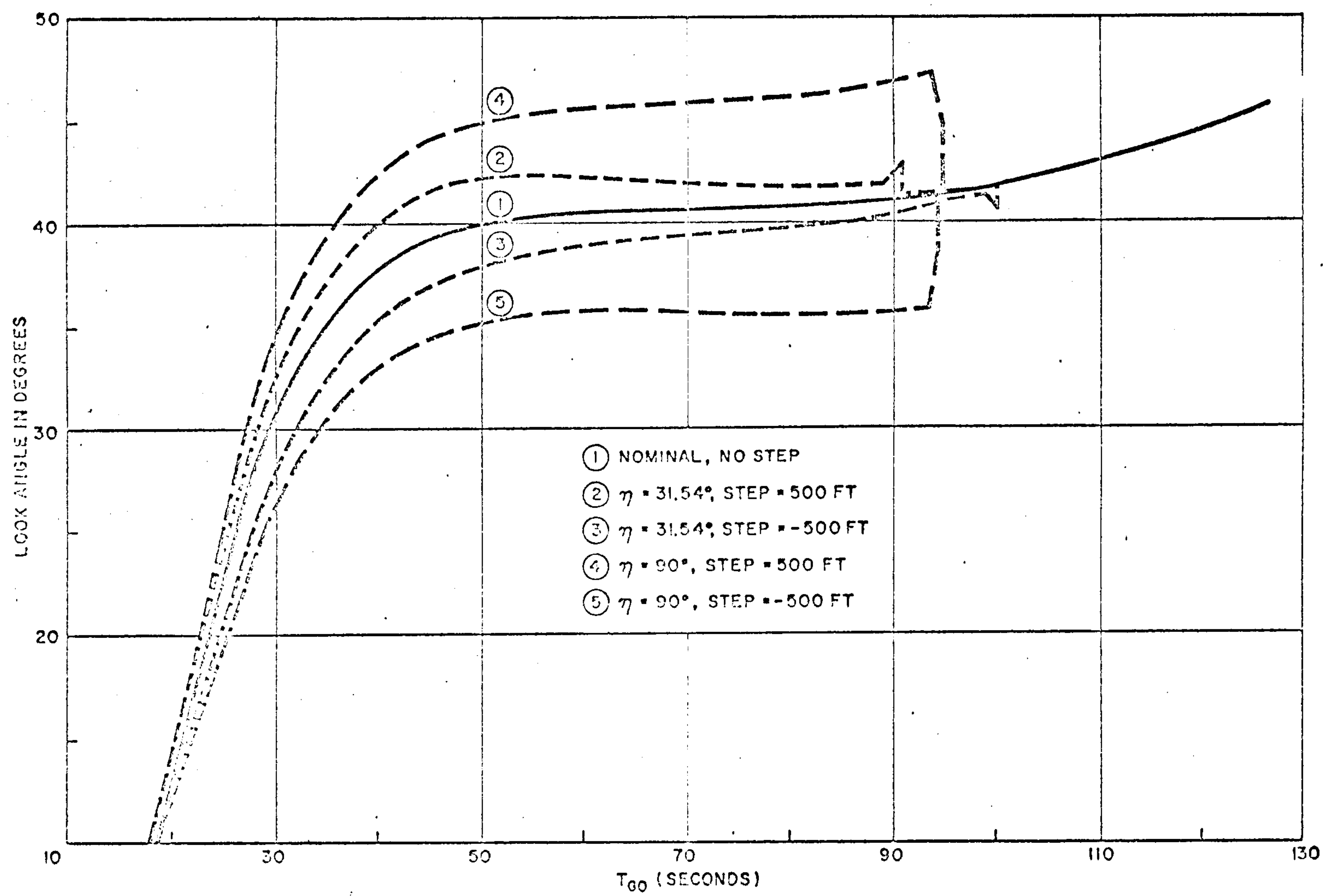

FIGURE 8. LOOK ANGLE VS TIME-TO-GO (STEP OF 500 FT AT A RANGE 10,000 FT FROM LANDING SITE).

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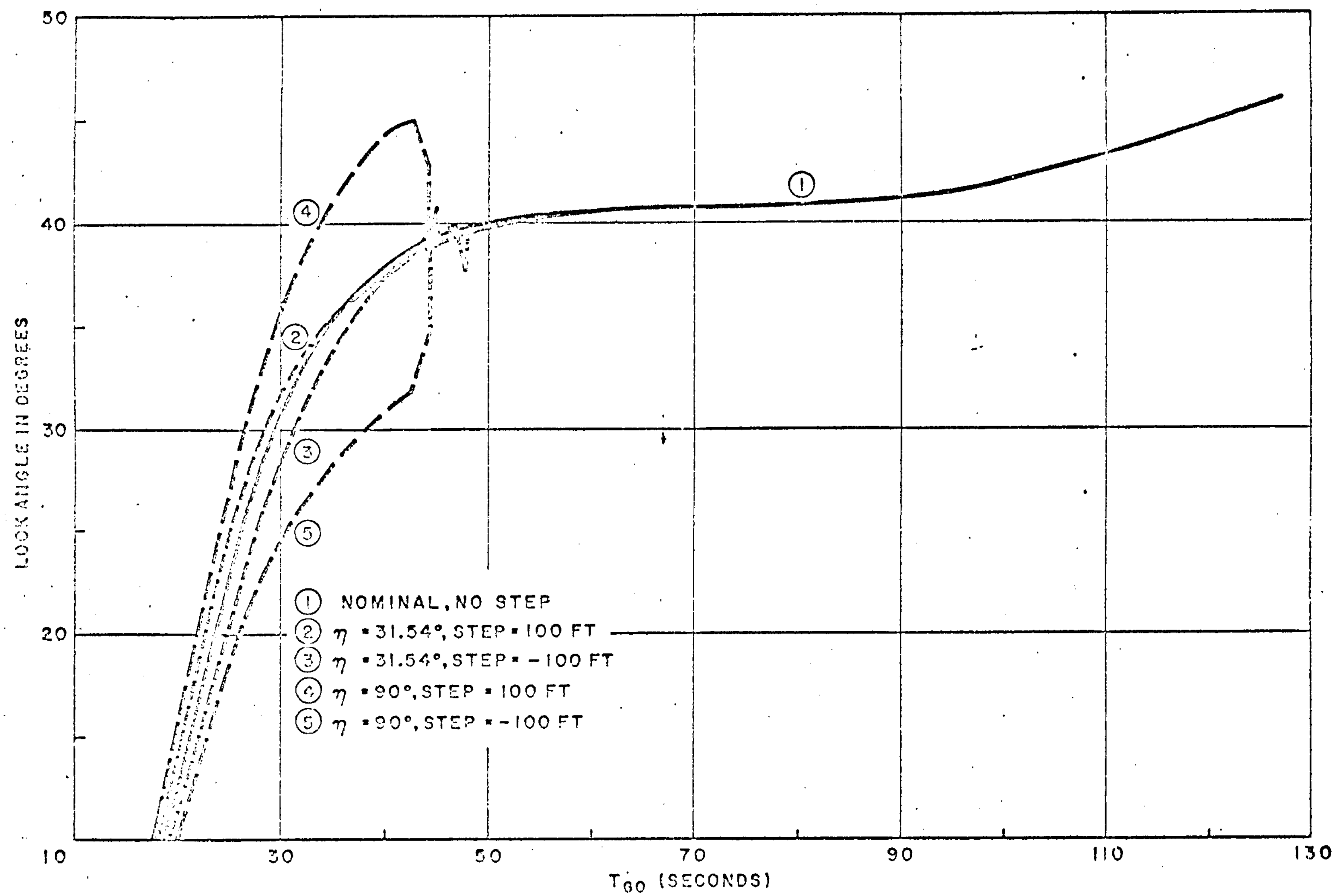

FIGURE 9. LOOK ANGLE VS TIME-TO-GO (STEP OF 100 FT AT AT A RANGE 1300 FT FROM LANDING SITE)

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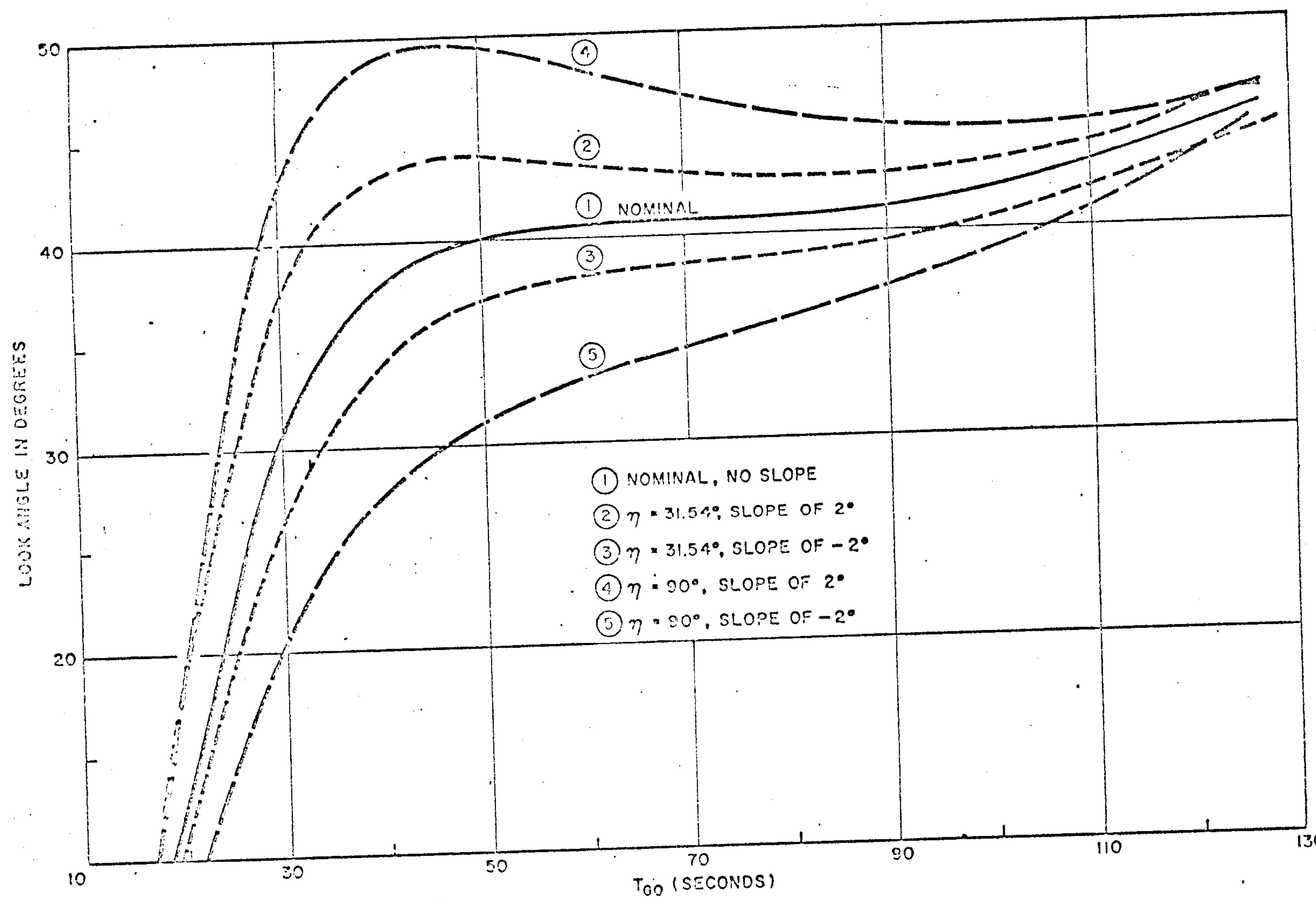

FIGURE 10. LOOK ANGLE VS TIME-TO-GO (SLOPE OF 2° IN TERRAIN PICKED UP BY RADAR 29,545 FT FROM LANDING SITE).

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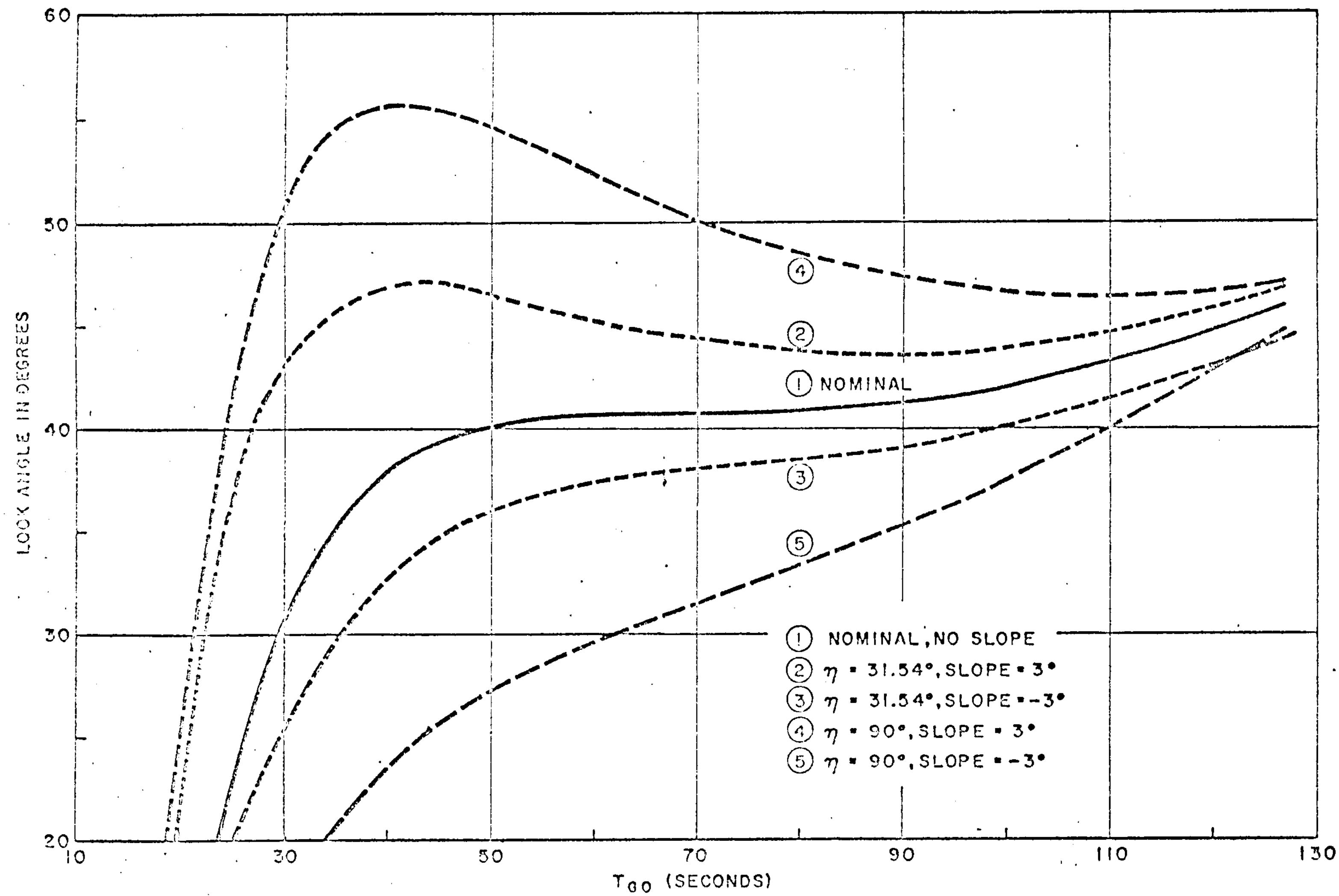

FIGURE 11. LOOK ANGLE VS TIME-TO-GO (SLOPE OF 3° PICKED UP BY RADAR 28,645 FT FROM LANDING SITE)

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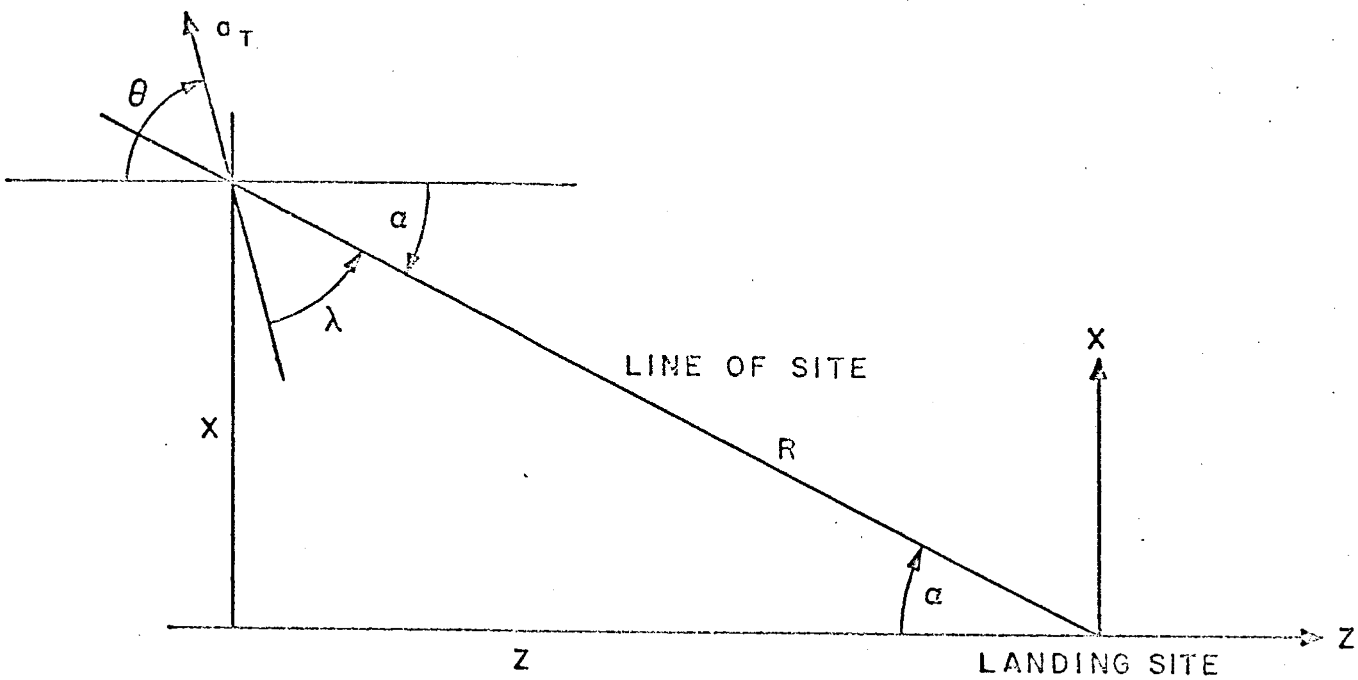

FIGURE 12. ANGLE RELATIONSHIPS